

Long-time Low-latency Quantum Memory by Dynamical Decoupling

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Practical quantum memory applications demand high-fidelity storage of arbitrary states for long times, while ensuring a small access latency and operating under technological control limitations. Building on exact results available for the paradigmatic case of a single qubit exposed to phase noise, we identify periodic repetition of a high-order dynamical decoupling sequence as a systematic strategy to meet these demands. We demonstrate how numerical search can be invoked to efficiently find the sequence best suited for repetition as well as for validating our analytical insights.

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The preservation of arbitrary quantum states – that is, quantum memory – in realistic, noisy physical systems is vital for quantum-enabled applications including secure communications, quantum computation and simulation, and quantum sensing and metrology. While numerous techniques relying on both open- and closed-loop control schemes have been devised in order to address this challenge, quantum error suppression strategies based on dynamical decoupling (DD) [1–4] are emerging as a method of choice for *physical-layer* decoherence control in non-Markovian open quantum systems. In particular, over the last few years, different multipulse DD protocols have been investigated in experimental qubit platforms as diverse as trapped ions [5], atomic ensembles [6], semiconductor quantum dots [7, 8], impurity spins in silicon [9], and nitrogen-vacancy centers in diamond [10].

The above investigations have consistently pointed to DD as a resource-efficient approach to substantially reducing physical error rates. However, they have largely focused on two control regimes [11]: the “coherence-time” regime, where the goal is to extend the characteristic (“ $1/e$ ” or T_2) decay time for coherence as long as possible, and the “high-fidelity” regime, where the goal is to suppress errors as low as possible for storage times short compared to T_2 (for instance, during a single quantum gate). Still, these two regimes do not capture the typical operating conditions of any true quantum memory, namely, error probabilities deep below the fault-tolerance threshold maintained for *arbitrarily long storage times*. This would be required, for instance, in a quantum repeater, or in a quantum computer where some quantum information must be maintained while large blocks of an algorithm are carried out on other qubits.

In this Letter, we show how DD can be employed to realize a useful quantum memory capable of preserving quantum information for long times with high fidelity, while meeting the practical requirement of a small *access latency*. We consider the simplest yet experimentally relevant setting of a single qubit undergoing phase decoherence and controlled through sequences of (ideally) instantaneous π pulses. We identify the *periodic repetition*

of a high-order DD sequence as an effective strategy for memory applications, and analytically characterize the resulting error rate and long-time coherence. This is accomplished by working within a filter-design framework which directly generalizes the transfer-function approach widely used across the engineering community [12] and provides a transparent picture of the controlled dynamics in the frequency domain, especially attractive for experimental implementations [5, 11, 13]. Through this approach, we identify conditions under which a “coherence plateau” can be engineered, and qubit fidelity *guaranteed* to a desired level at long storage times. Finally, we use numerical search within the practically important family of *digital* DD sequences to both find optimal sequences in the long-time limit, and to verify that a periodic structure naturally emerges in the best performing sequences at sufficiently long storage times.

DD for a single qubit undergoing phase decoherence due to the interaction with a quantum bath or, in the semiclassical regime, a stochastic field along the (z) quantization axis, has been extensively discussed in the literature [1, 4, 14–18]. Thus, we only summarize key concepts and notation. Each control operation effects an instantaneous π rotation around the x axis, and the DD sequence may be specified in terms of the pulse timings $\{t_j\}_{j=1}^n$, where we also define $t_0 \equiv 0$, and $t_{n+1} \equiv T_p$ as the sequence duration. We take all the interpulse intervals ($t_{j+1} - t_j$) to be lower-bounded by a *minimum interval* τ [17]. DD implements the control propagator $U_c(t) = \sigma_x^{[y_p(t)+1]/2}$, where the sequence propagator $y_p(t)$ is a piecewise-constant bi-valued function that switches between ± 1 whenever a pulse is applied [19].

The effect of DD on qubit dephasing may be evaluated exactly via calculation of a spectral overlap of the control modulation and the noise power spectral density, $S(\omega) \equiv S(-\omega)$, which is determined by the Fourier transform of the two-time noise correlation function. This is true so long as the environment can be approximated as a bosonic bath in thermal equilibrium or the fluctuations are a stationary Gaussian process. In most physical scenarios of interest, $S(\omega)$ has a power-law behavior

at low frequencies, and decays to zero beyond an upper cutoff $\omega_c > 0$, that is, $S(\omega) \propto |\omega|^s f(|\omega|, \omega_c)$, and the “rolloff function” f specifies the high-frequency behavior, $f = \Theta(|\omega| - \omega_c)$ corresponding to a “hard” cutoff. Let $\tilde{y}_p(\omega)$ denote the Fourier transform of $y_p(t)$, which can be written as $\tilde{y}_p(\omega) = \omega^{-1} \sum_{j=0}^n (-1)^j [\exp(it_j \omega) - \exp(it_{j+1} \omega)]$ [4, 15]. The *filter function* of the sequence p is then given by $F_p(\omega) = \omega^2 |\tilde{y}_p(\omega)|^2$, and the controlled qubit coherence decays over time as $e^{-\chi_p}$, where the *decoupling error* χ_p at time $t = T_p$ is

$$\chi_p = \int_0^\infty \frac{S(\omega)}{2\pi\omega^2} F_p(\omega) d\omega. \quad (1)$$

The case $n = 0$ recovers free evolution over $[0, T_p]$. For any sequence p , the *order of error suppression* α_p is determined by the scaling of the filter function near $\omega = 0$, that is, $F_p(\omega) \propto (\omega\tau)^{2(\alpha_p+1)}$. A high multiplicity of the zero at $\omega = 0$ ultimately leads to a perturbatively small value of χ_p as long as $\omega_c\tau \ll 1$.

Achieving low-error over a desired storage time T_s is straightforward in principle if arbitrarily fast control ($\tau \rightarrow 0$) is allowed, since it suffices to use a high-order DD sequence such as Concatenated DD (CDD, [3]) or Uhrig DD (UDD, [4]) over $T_s \equiv T_p$. For instance, the UDD $_n$ sequence achieves $\alpha_p = n$ with n pulses, applied at $t_j = T_p \sin^2[\pi j/(2n+2)]$. Increasing α_p implies increasing n , at the expenses of shrinking the minimum control time scale as $\tau \equiv t_1 = \mathcal{O}(T_p/n^2)$ [17].

Moving from this idealized scenario to a practically useful, long-time quantum memory requires several linked issues to be considered: (i) Perturbative DD sequences are *not* generally viable for high-fidelity long-time storage. Once the constraint $\tau > 0$ is enforced, increasing α_p necessitates extension of T_p . As T_p grows, however, the underlying perturbative corrections eventually catch up, and a *maximal* duration emerges beyond which further increasing α_p will *increase* the total error [17, 20]. (ii) Numerical DD approaches such as randomized [21] or optimized Bandwidth-Adapted DD (BADD [17]), which can in principle provide low error at long times, become impractical as the configuration space of *all* possible DD sequences over which to search grows exponentially with T_s . Simplified sequence generation approaches are therefore required. (iii) A stored quantum state must be accessible not just at a designated retrieval time but *throughout* a time span during which it might serve as an input to a quantum protocol. Since, however, DD exploits interference pathways between control-modulated trajectories, mid-sequence interruption ($t < T_p$) may result in significantly sub-optimal performance (Fig. 1). Reducing access-latency well below the maximum storage time T_s , while maintaining low error rates, must therefore be considered in designing DD approaches to quantum memory. (iv) In real settings, the practical challenges posed by sequencing complexity should be minimized while maintaining

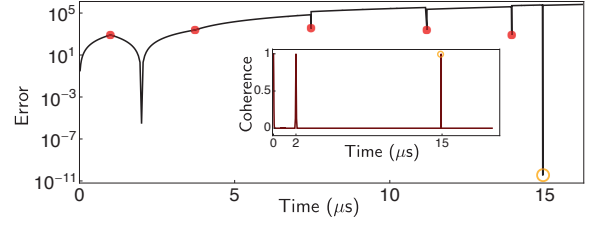


FIG. 1: (Color online) Decoupling error and coherence (inset) during a UDD $_5$ sequence with minimum interpulse time $\tau = 1 \mu\text{s}$. Pulse times are marked with filled circles while the open circle indicates the readout time T_p . Minimal error (maximal coherence) is reached only at the conclusion of the sequence, with the coherence spike near $2 \mu\text{s}$ resulting from a spin-echo effect. Throughout this work, we assume a phenomenological noise model appropriate for a singlet-triplet spin qubit in GaAs, $S(\omega) = \alpha\omega_c(\omega/\omega_c)^{-2}e^{-\omega^2/\omega_c^2}$, with $\omega \in [\omega_{\min}, \omega_{\max}]$. We set $\alpha = 0.207$, $\omega_c/2\pi = 10\text{kHz}$, $\omega_{\min}/2\pi = 0.01\text{Hz}$, and $\omega_{\max}/2\pi = 10^8\text{Hz}$ to maximize agreement with the measured T_2 ($\approx 35\text{ ns}$) [7, 22]. We chose τ well above technological constraints ($\sim \text{ns}$) in order to reduce n .

performance. Since the minimum time scale τ typically is set by the clock speed of the electronic hardware, this mandates *digital modulation*, whereby all interpulse intervals are constrained to be integer multiples of τ [18].

Addressing all such issues simultaneously motivates a systematic “modular” approach to generate low-error DD sequences for long-time storage out of shorter blocks. We identify *periodic repetition* of a base, high-order DD cycle as a natural strategy towards meeting our goal. In fact, this general approach is widely used implicitly, *e.g.*, the well-known Carr-Purcell (CP) sequence $\{\tau, 2\tau, 2\tau, \dots, 2\tau, \tau\}$ is generated by repeating the sequence $\{\tau, 2\tau, \tau\}$. In order to develop an analytical understanding of the effects of repeating an *arbitrary* DD sequence, we observe that filter functions have simple transformation properties under sequence combination. Imagine that two sequences, p_1 and p_2 , are joined to form a longer one, denoted $p_1 + p_2$, with a propagator $y_{p_1+p_2}(t)$ that is made by continuously attaching each sequence propagator. In the Fourier space we have

$$\tilde{y}_{p_1+p_2}(\omega) = \tilde{y}_{p_1}(\omega) + e^{i\omega T_{p_1}} \tilde{y}_{p_2}(\omega). \quad (2)$$

Consider now repeating a sequence p , of duration T_p , m times, and let $[p]^m$ denote the resulting sequence of duration $T_s = mT_p$. By iterating Eq. (2), the following exact expression is found for the DD error:

$$\chi_{[p]^m} = \int_0^\infty \frac{S(\omega)}{2\pi\omega^2} \frac{\sin^2(m\omega T_p/2)}{\sin^2(\omega T_p/2)} F_p(\omega) d\omega. \quad (3)$$

Eq. (3) describes dephasing dynamics under *arbitrary* periodic multipulse control, generalizing the special case of Periodic DD (PDD), $p = \{\tau, \tau\}$ [16]: the single-cycle filter function $F_p(\omega)$ is multiplied by a factor which is rapidly oscillating for large m and develops peaks scaling

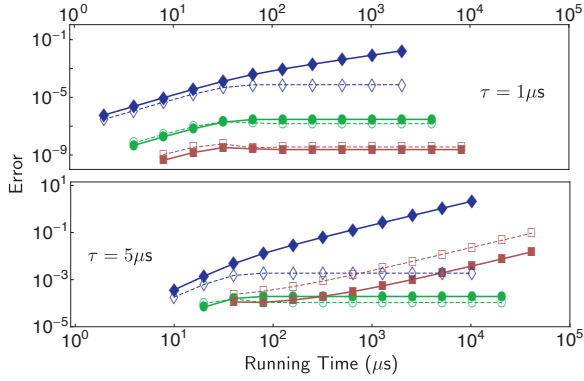


FIG. 2: (Color online) DD error for repetition of PDD(\diamond), CP(\circ), and CDD₃(\square) sequences ($\alpha_p = 1, 2, 3$, respectively). $S(\omega) \propto \omega^s$, with $s = -2$ (solid) and $s = 0$ (dashed). The minimum interpulse interval τ is indicated in each panel.

with $\mathcal{O}(m^2)$ at multiples of the “resonance frequency” $\omega_{\text{res}} = 2\pi/T_p$, introduced by the periodic modulation.

After many repeats, the DD error is determined by the interplay between the order of error suppression of the base sequence ($F_p(\omega) \propto \omega^{2(\alpha_p+1)}$), the noise power behavior at low frequencies ($S(\omega) \propto \omega^s$), and the size of noise contributions at the resonance frequencies. The situation is simplest in the case of a hard upper frequency cutoff at ω_c . Using the Riemann-Lebesgue lemma, the oscillating factor may be removed, resulting in the following asymptotic expression:

$$\lim_{m \rightarrow \infty} \chi_{[p]}^m \equiv \chi_{[p]}^\infty = \int_0^{\omega_c} \frac{S(\omega)}{4\pi\omega^2} \frac{F_p(\omega)}{\sin^2(\omega T_p/2)} d\omega, \quad (4)$$

provided that $\chi_{[p]}^\infty$ is finite. To visualize, the DD error initially increases as $(m^2\chi_p)$, until coherence saturates to a non-zero *residual plateau* at $e^{-\chi_{[p]}^\infty}$, and *no further decoherence occurs* [23].

The emergence of coherence plateau after repeating DD sequences can *guarantee* high fidelity throughout long storage times with a latency $t_\ell = T_p \ll T_s$. Since, by comparing Eqs. (3)-(4), it follows that $\chi_{[p]}^m \leq 2\chi_{[p]}^\infty$ for all m , it suffices to pick a base sequence with a sufficiently small residual error $\chi_{[p]}^\infty$. Mathematically, the existence of such a coherence guarantee requires that the following conditions be simultaneously obeyed:

$$s + 2\alpha_p > 1, \quad T_p\omega_c < 2\pi, \quad (5)$$

which corresponds, respectively, to removing the possible singularity of the integrand at 0 or ω_{res} in Eq. (4). Thus, judicious selection of a base sequence, fixing α_p and T_p , can guarantee saturation of coherence in principle.

In a realistic scenario where the noise spectrum does not have a hard cutoff (*e.g.*, where a residual noise-floor persists beyond ω_c), it is impossible to fully avoid the singular behavior introduced by the periodic modulation at high frequencies as $m \rightarrow \infty$. Contributions from the resonating region $\omega \approx \omega_{\text{res}}$ are amplified with m , causing the

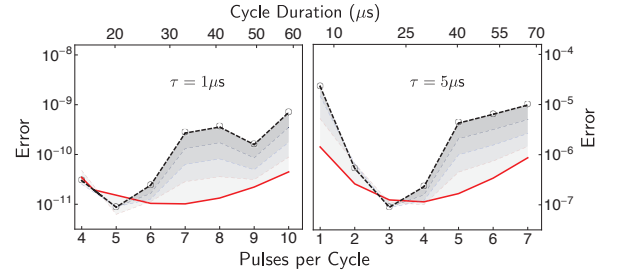


FIG. 3: (Color online) DD error for repetition of UDD sequences. The sequences are labeled by both their pulse number and duration. The solid series depicts the single cycle errors whilst the dashed lines represent sequences repeated 2, 4, 8, and 16 times (with increasingly dark shading). The minimum-error UDD sequences for multiple repeats vs. single cycle are distinct, *i.e.*, UDD₅ vs. UDD₇ (UDD₃ vs. UDD₄) in the left (right) panel.

error to increase unboundedly with time and coherence to ultimately decay to zero. Nonetheless, a very large number m of repetitions may still be applied before the resonating contributions become important. This makes it possible to engineer a coherence plateau over an intermediate time regime which can be very long from a practical standpoint, depending on how fast $S(\omega)$ decays beyond ω_c . Fig. 2 illustrates the emergence of this coherence plateau based on the order of error suppression, α_p , and noise power law s , as well as (lower panel) the exit from the plateau due to the relatively soft Gaussian rolloff as a larger τ (and subsequently T_p) is used and the condition set by Eq. (5) is no longer met.

Finding the optimal base-sequence for repetition can still be challenging and non-intuitive. For example, the sequences best suited for long-time storage need *not* coincide with those optimizing short-time performance. In Fig. 3 we compare calculated error rates for single cycle and multiple repetitions of UDD sequences. The local minima indicate that the sequence with minimal error at a single repeat is different from the sequence with minimal error at many repetitions. In general, numerical search is necessary to find the optimal base sequence for repetition [17], with a sufficiently small latency, $t_\ell = T_p \ll T_s$, to meet system constraints. A main advantage compared with a direct optimization over all possible sequences running up to T_s (BADD) is that searching for a low-error sequence with a duration constrained to $2\pi/\omega_c$, that will be repeated up to T_s , is far more tractable computationally.

Remarkably, a periodic structure for low-error long-time DD still *naturally emerges* through a direct BADD search up to time T_s , when additional *sequencing constraints* are enforced. We turn to the Walsh DD (WDD) family of sequences to employ a digital modulation that is efficient in the complexity of sequencing [18]. Starting with a free evolution of duration τ , all possible WDD sequences can be recursively built out of simpler WDD

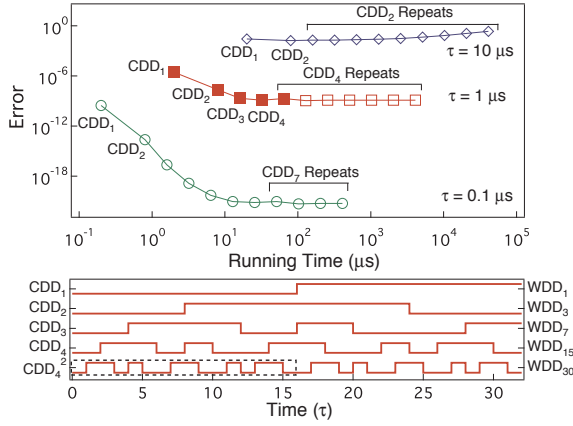


FIG. 4: (Color online) Top: Minimal-error DD sequences from numerical search over WDD sequences, for $\tau = 0.1, 1, 10\mu\text{s}$. In each series, the minimal-error sequences systematically access higher orders of error cancellation (via concatenation) over increasing running times, until an optimal concatenated sequence is found which is then repeated in the longer minimal-error sequences. Note that spin-echo, CPMG, CDD and its relatives appear naturally in the WDD basis [18]. The gradual increase in error for the series with $\tau = 10\mu\text{s}$ (loss of coherence plateau) is due to the softness of the high-frequency cut-off. Bottom: Control propagators corresponding to the solid markers in the middle data series ($\tau = 1\mu\text{s}$), showing the emergence of a periodic structure for sufficiently long storage time. Control propagators scaled to same length for ease of comparison. Dashed box highlights base sequence CDD_4 that is repeated for long times.

ones. In fact, there are $\frac{1}{2}(T_s/\tau)$ WDD sequences that stop at time T_s , a very small subset of all $2^{T_s/\tau}$ possible digital DD sequences, enabling an otherwise intractable numerical BADD search up to T_s using Eq. (1). Representative results are shown in Fig. 4, where for each T_s all WDD sequences with given τ are evaluated and those with the lowest residual error are selected. As T_s grows, minimal-error sequences (shown) are found to consist of a few concatenation steps (increasing α_p of the base sequence) followed by successive repetitions of that fixed base sequence. Accordingly, *mid-sequence interrupts at times corresponding to multiples of T_p are permitted in low-error long-time storage scenarios without substantial performance degradation*. In addition to providing a direct means of finding optimal long-time DD sequences, these numerical search results thus support our key analytical insights as to use of periodic sequences.

While our analysis has assumed perfect π rotations, various pulse non-idealities are inevitably present in practice, their accumulated effect potentially resulting in severe performance degradation. Beside minimizing the number of applied pulses (thus using longer intervals as noted), several approaches can be envisioned to mitigate the impact of the dominant pulse imperfections. In particular, Euler DD cycles [2] may provide a starting

point to compensate for finite-width errors, whereas the sensitivity of DD to systematic pulse imperfections can be largely eliminated by incorporating robust composite pulses or symmetric self-correcting cycles, as successfully demonstrated in recent DD nuclear magnetic resonance experiments with long pulse trains [24].

In summary, we have addressed a timely and important problem in quantum information – determining a means to effectively produce a practically useful quantum memory via DD. We have identified the key requirements towards this end, and developed a strategy for sequence construction based on repetition of high-order base sequences. Our results allow analytical bounding of the long-time error rates and identify conditions in which a maximum error rate can be *guaranteed* for long times. Finally, we have validated these insights and analytic calculations using an efficient search over WDD sequences under realistic noise conditions. Based on these results, we expect that repetition of suitably designed DD cycles will provide a practical avenue to high-fidelity low-latency quantum storage.

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- [1] L. Viola and S. Lloyd, Phys. Rev. A **58**, 2733 (1998); L. Viola, E. Knill, and S. Lloyd, Phys. Rev. Lett. **82**, 2417 (1999).
 - [2] L. Viola and E. Knill, Phys. Rev. Lett. **90**, 037901 (2003).
 - [3] K. Khodjasteh and D. A. Lidar, Phys. Rev. Lett. **95**, 180501 (2005).
 - [4] G. S. Uhrig, Phys. Rev. Lett. **98**, 100504 (2007).
 - [5] M. J. Biercuk *et al.*, Nature **458**, 996 (2009); Phys. Rev. A **79**, 062324 (2009); H. Uys, M. J. Biercuk, and J. J. Bollinger, Phys. Rev. Lett. **103**, 040501 (2009).
 - [6] Y. Sagi, I. Almog, and N. Davidson, Phys. Rev. Lett. **105**, 053201 (2010); I. Almog *et al.*, J. Mod. Phys. **44**, 154006 (2011).
 - [7] S. Foletti *et al.*, Nature Phys. **5**, 903 (2009); H. Bluhm *et al.*, *ibid.* **7**, 109 (2011); M. D. Schulman *et al.*, arXiv:1202.1828.
 - [8] C. Barthel *et al.*, Phys. Rev. Lett. **105**, 266808 (2010); J. Medford *et al.*, arXiv:1108.3682.
 - [9] A. M. Tyryshkin *et al.*, arXiv:1011.1903; Z.-H. Wang *et al.*, arXiv:1011.6417.
 - [10] G. de Lange *et al.*, Science **330**, 60 (2010); C. A. Ryan, J. S. Hodges, and D. G. Cory, Phys. Rev. Lett. **105**, 200402 (2010); B. Naydenov *et al.*, Phys. Rev. B **83**, 081201(R) (2011); Z.-H. Wang *et al.*, arXiv:1202.0462.
 - [11] M. J. Biercuk, A. Doherty, and H. Uys, J. Phys. B **44**, 154002 (2011).
 - [12] J. Rutman, Proc. IEEE **66**, 1048 (1978).
 - [13] T. Green, H. Uys, and M. J. Biercuk, arXiv:1110.6686.
 - [14] M. Palma, K.-A. Suominen, and A. K. Ekert, Proc. R.

- Soc. London A **452**, 567 (1996).
- [15] L. Cywinski *et al.*, Phys. Rev. B **77**, 174509 (2008).
 - [16] T. Hodgson, L. Viola, and I. D’Amico, Phys. Rev. A **81**, 062321 (2010).
 - [17] K. Khodjasteh, T. Erdélyi, and L. Viola, Phys. Rev. A **83**, 020305 (2011).
 - [18] D. Hayes *et al.*, Phys. Rev. A **84**, 062323 (2011).
 - [19] All sequence propagators are taken to begin and end at 1, requiring a pulse to be applied at $t = T_p$ if n is odd.
 - [20] G. S. Uhrig and D. A. Lidar, Phys. Rev. A **82**, 012301 (2010).
 - [21] L. Viola and E. Knill, Phys. Rev. Lett. **94**, 060502 (2005); L. F. Santos and L. Viola, *ibid.* **97**, 150501 (2006).
 - [22] M. J. Biercuk and H. Bluhm, Phys. Rev. B **83**, 235316 (2011).
 - [23] This phenomenon is related to observations in precision oscillator characterization noting that in certain circumstances phase noise saturates at long times, given a sharp noise cutoff [12]. The possibility of residual quantum coherence is known to occur for *free supraohmic dephasing* dynamics [14, 16]. For dephasing under *arbitrary* periodic DD, our results imply that asymptotic residual coherence can strictly exist only for a hard spectral cutoff.
 - [24] A. M. Souza, G. A. Álvarez, and D. Suter, Phys. Rev. Lett. **106**, 240501 (2011); arXiv:1110.6334.